

Operation Transforms		
N	F(s)	f(t), t > 0
1.1	$Y(s) = \int_0^{\infty} \exp(-st)y(t)dt$	definition of a Laplace transform $y(t)$
1.2	$Y(s)$	inversion formula $y(t) = \frac{1}{j2\pi} \int_{c-j\infty}^{c+j\infty} \exp(st)Y(s) ds$
1.3	$sY(s) - y(0)$	first derivative $y'(t)$
1.4	$s^2 Y(s) - sy(0) - y'(0)$	second derivative $y''(t)$
1.5	$s^n Y(s) - s^{n-1}[y(0)]$ $- s^{n-2}[y'(0)] - \dots - s[y^{(n-2)}(0)]$ $- [y^{(n-1)}(0)]$	nth derivative $y^{(n)}(t)$
1.6	$\frac{1}{s}F(s)$	integration $\int_0^t Y(\tau) d\tau$
1.7	$F(s)G(s)$	convolution integral $\int_0^t f(t-\tau)g(\tau)d\tau$
1.8	$\frac{1}{\alpha}F\left(\frac{s}{\alpha}\right)$	$f(\alpha t)$
1.9	$F(s - \alpha)$	shifting in the s-plane $\exp(\alpha t) f(t)$
1.10	$\frac{1}{1 - \exp(-sT)} \int_0^T \exp(-st)f(t) dt$	f(t) has period T, such that $f(t+T) = f(t)$
1.11	$\frac{1}{1 + \exp(-sT)} \int_0^T \exp(-st)g(t) dt$	g(t) has period T, such that $g(t+T) = -g(t)$
Function Transforms		

N	F(s)	f(t), t > 0
---	------	-------------

2.1	1	$\delta(t)$, unit impulse at $t = 0$
2.2	s	$\frac{d}{dt} \delta(t)$, double impulse at $t = 0$
2.3	$\exp(-\alpha s), \alpha \geq 0$	$\delta(t - \alpha)$
2.4a	$\frac{1}{s}$	unit step $u(t)$
2.4b	$\frac{1}{s} [\exp(-as) - \exp(-bs)]$	$\begin{array}{ll} 0 & t < a \\ 1 & a < t < b \\ 0 & t > b \end{array}$
2.5	$\frac{1}{s} \exp(-\alpha s)$	$u(t - \alpha)$
2.6	$\frac{1}{s^2}$	t
2.7a	$\frac{1}{s^n}, n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!}$
2.7b	$\frac{n!}{s^{n+1}}, n = 1, 2, 3, \dots$	t^n
2.8	$\frac{1}{s^k}, k$ is any real number > 0	$\frac{t^{k-1}}{\Gamma(k)}$ the Gamma function is given in Appendix A
2.9	$\frac{1}{s + \alpha}$	$\exp(-\alpha t)$
2.10	$\frac{1}{(s + \alpha)^2}$	$t \exp(-\alpha t)$

2.11	$\frac{1}{(s + \alpha)^n}, n=1, 2, 3, \dots$	$\left[\frac{t^{n-1}}{(n-1)!} \right] \exp(-\alpha t)$
2.12	$\frac{\alpha}{s(s + \alpha)}$	$1 - \exp(-\alpha t)$
2.13	$\frac{1}{(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{(\beta - \alpha)} [\exp(-\alpha t) - \exp(-\beta t)]$
2.14	$\frac{1}{s(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{\alpha\beta} + \frac{\exp(-\alpha t)}{\alpha(\alpha - \beta)} + \frac{\exp(-\beta t)}{\beta(\beta - \alpha)}$
2.15	$\frac{s}{(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{(\alpha - \beta)} [\alpha \exp(-\alpha t) - \beta \exp(-\beta t)]$
2.16a	$\frac{\alpha}{s^2 + \alpha^2}$	$\sin(\alpha t)$
2.16b	$\frac{[\sin(\phi)]s + \alpha[\cos(\phi)]}{s^2 + \alpha^2}$	$\sin(\alpha t + \phi)$
2.17	$\frac{s}{s^2 + \alpha^2}$	$\cos(\alpha t)$
2.18	$\frac{s^2 - \alpha^2}{[s^2 + \alpha^2]^2}$	$t \cos(\alpha t)$
2.19	$\frac{1}{s(s^2 + \alpha^2)}$	$\frac{1}{\alpha^2} [1 - \cos(\alpha t)]$
2.20	$\frac{1}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha^3} [\sin(\alpha t) - \alpha t \cos(\alpha t)]$
2.21	$\frac{s}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha} [t \sin(\alpha t)]$
2.22	$\frac{s^2}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha} [\sin(\alpha t) + \alpha t \cos(\alpha t)]$

2.23	$\frac{1}{(s^2 + \omega^2)(s^2 + \alpha^2)}, \alpha \neq \omega$	$\left\{ \frac{1}{\omega^2 - \alpha^2} \right\} \left\{ \frac{1}{\alpha} \sin(\alpha t) - \frac{1}{\omega} \sin(\omega t) \right\}$
2.24	$\frac{\alpha}{s^2(s + \alpha)}$	$t - \frac{1}{\alpha} [1 - \exp(-\alpha t)]$
2.25	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \sin(\beta t)$
2.26	$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \cos(\beta t)$
2.27	$\frac{s + \lambda}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \left\{ \cos(\beta t) + \left[\frac{\lambda - \alpha}{\beta} \right] \sin(\beta t) \right\}$
2.28	$\frac{s + \alpha}{s^2 + \beta^2}$	$\frac{\sqrt{\alpha^2 + \beta^2}}{\beta} \sin(\beta t + \phi), \phi = \arctan\left(\frac{\beta}{\alpha}\right)$
2.29	$\frac{1}{s^2 - \alpha^2}$	$\frac{1}{\alpha} \sinh(\alpha t)$
2.30	$\frac{s}{s^2 - \alpha^2}$	$\cosh(\alpha t)$
2.31	$\arctan\left(\frac{\alpha}{s}\right)$	$\frac{1}{t} \sin(\alpha t)$
2.32	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
2.33	$\frac{1}{\sqrt{s + \alpha}}$	$\frac{1}{\sqrt{\pi t}} \exp[-\alpha t]$
2.34	$\frac{1}{\sqrt{s^3}}$	$2 \sqrt{\frac{t}{\pi}}$
2.35	$\frac{1}{\sqrt{s^2 + \alpha^2}}$	$J_0(\alpha t)$
		Bessel function given in
2.36	$\frac{1}{(s^2 + \alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right) J_1(\alpha t)$

2.37	$\frac{1}{\sqrt{s^2 - \alpha^2}}$	$I_0(\alpha t)$ Modified Bessel function
2.38	$\frac{1}{(s^2 - \alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right) I_1(\alpha t)$
2.39	$\sqrt{s - \alpha} - \sqrt{s - \beta}$	$\frac{1}{2t\sqrt{\pi t}} [\exp(\beta t) - \exp(\alpha t)]$